147. Proposed by R. A. WELLS, Professor of Mathematics, Franklin College, Athens, Ohio.

Find the locus in space of the point which is equally illuminated by each of two unequal lights whose intensities are a and b ($a>b$), placed at a distance c from each other.

Solution by WILLIAM HOOVER, A.M., Ph.D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let (x, y, z) be the rectangular coördinates of any one of the illuminated points, the mid-point of the line joining the two lights taken along the x-axis as origin; then the coördinates of the two lights may be given as (l, 0, 0), ($-l$, 0, 0), and by theory

$$\frac{a}{(x-l)^2+y^2+z^2}=\frac{b}{(x+l)^2+y^2+z^2}\ldots.(1),$$

or $x^2+y^2+z^2+2l\dfrac{a+b}{a-b}x+l^2=0\ldots.(2)$, a sphere.

Also solved by *G. B. M. ZERR*, and *J. SCHEFFER*.

148. Proposed by DR. E. D. ROE, JR., Associate Professor of Mathematics in Syracuse University, Syracuse, N. Y.

The condition that two triangles, abc, xyz, are similar is

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ x & y & z \end{vmatrix}=0,$$

and the condition that the triangle abc is equilateral is

$$\begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1 \end{vmatrix}=0.$$

(Used in solving 130.)

I. Solution by the PROPOSER.

First. *The given conditions are necessary*, for if two triangles abc, xyz, be similar, then $\dfrac{a-b}{x-y}=\dfrac{b-c}{y-z}=\dfrac{c-a}{z-x}=r\ldots.(1)$, for this expresses not only that the homologous sides are proportional, but also that the homologous angles are equal. (1) is equivalent to the equations

$$\left.\begin{aligned} a-b&=r(x-y) \\ b-c&=r(y-z) \\ c-a&=r(z-x) \end{aligned}\right\}\ldots(2).$$

Multiplying the equations (2) by x, y, and z, respectively, and adding, we get

$$\Sigma(a-b)z=r\Sigma(x-y)z=0\ldots.(3),$$

or $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ x & y & z \end{vmatrix}=0\ldots.(4)$, which is therefore necessary at least, if the triangles are similar.